Class I: Introduction + Measures of Location

Monday
June 24, 2024

Paper 3: - Quantitative Methods (Q.M)
Techniques (Q.T)
- Business statistics

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Participants' Expectations

- ✓ More of calculations
- ✓ A lot of methods
- ✓ A lot of formulars
- ✓ Full of statistics
- ✓ Paper being hard
- ✓ Graphs
- ✓ Dealing with BIG figures
- ✓ Pure mathematics
- ✓ Using Log books - Table
- ✓ Twisted Questions / Tricky
- ✓ Logical Questions
- ✓ Limited choice of Questions
- ✓ Limited time with longer arithmetic

- ✓ Lot of notes
  - Hard to complete the syllabus.

3 ½ months

ABE
✓
✓

My Expectation(s)

mon – Thu m.

4pa

- ✓ PASSING
- ✓ COMMITMENT

Papers Quantitative meth

Quantitative
- ✓ compute
- ✓ Graph
- ✓ Read Tables

Qualitative
- ✓ Define
- ✓ Outline
- ✓ Interpret.

Measures of Location

✓ A measure of location is value to which a given data set tends to concentrate; It is also called the measure of central Tendency.

- Place
- Point in time
- Area

✓ Measures of location include:

- The mean
- The Mode
- The Median
- The Quantiles
  - ✓ Quartiles
  - ✓ Deciles
  - ✓ Percentiles

The Mean

✓ It is defined to be the average of a given data set; The mean may be categorised into:

- The arithmetic mean
- The weighted mean
- The Geometric mean
- The Harmonic mean

① The Arithmetic mean

Case I: For ungrouped Data:

✓ Given a set of data x_i, the arithmetic mean $\bar{x}$, is given by:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{N}$$

i = position

x $\quad x_i$ $\qquad \sum_{i=1}^{n} x_i$

where;

x_i = data set values

N = number of data values.

$\sum$ = sigma notation for "sum of"

Example:

The revenue (US$) generated by W-Inc were as noted:

Month	Jan	Feb	mar	April	May
Revenue (US$)	600	550	650	450	750

Determine the average revenue using the arithmetic mean.

Recall:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{N} \quad ; \quad i = 3$$

$x_i = \underline{\underline{650}}$

$\hookrightarrow x_3$

$x_i = \{600, 550, 650, 450, 750\}$

$$\bar{x} = \frac{600 + 550 + 650 + 450 + 750}{5} = \frac{3000}{5}$$

$$\underline{\bar{x} = 600 \text{ USD}}$$

Monday
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Class II

NB: In some instances a data value may repeat; we shall generate a frequency distribution and then apply the formular:

$$\bar{x} = \frac{\sum_{i=1}^{n} (f_i \cdot x_i)}{\sum_{i=1}^{n} f_i}$$

where:

f_i = frequency of the data set values

x_i = data set values

Example:

The scores of some students that sat exam A were as noted:

$\cancel{60}$ $\cancel{80}$ $\cancel{75}$ $\cancel{60}$ $\cancel{85}$

$\cancel{70}$ $\cancel{75}$ $\cancel{60}$ $\cancel{80}$ $\cancel{90}$

$60 + 60 + 60 + 60$

$60 + 60 = \boxed{60 \times 6}$

$\quad\quad\quad\quad\quad x \cdot f$

~~95~~ ~~90~~ ~~70~~ ~~60~~ ~~55~~

~~90~~ ~~55~~ ~~60~~ ~~95~~ ~~90~~

~~60~~ ~~85~~ ~~75~~ ~~90~~ ~~70~~

60×6

Recall:

$$\bar{x} = \frac{\sum_{i=1}^{n}(f_i \cdot x_i)}{\sum_{i=1}^{n} f_i}$$

Determine the average score.

Score(x)	Tally	f_i *	$f_i \cdot x_i$
60	$\cancel{||||}$ |	6	60+60+6 360 → 6×60
70	||| ||	3	210
95	||	2	190
90	$\cancel{||||}$	5	450
80	||	2	160
75	|||	3	225
55	||	2	110
85	||	2	170
	Total 25	25	1875

$$\bar{x} = \frac{1875}{25} = 75$$

$$\bar{x} = \underline{\underline{75}}$$

Case II: For Grouped Data

Once the class mark (mid-value) has been obtained, we apply the formular:

$$\bar{x} = \frac{\sum_{i=1}^{n}(f_i \cdot x_i)}{\quad}$$ ✓✓

$$\sum_{i=1}^{n} f_i$$

where;

f_i = class frequency

x_i = class mark

ie $x_i = \dfrac{\text{Lower class limit / boundary + upper class limit / boundary}}{2}$

Example:

The costs of production of 20 units in J-Inc were as noted.

Costs'00 USD	60 - 80	80 - 100	100 - 120	120 - 140	140 - 200
units	4	2	8	3	3

Determine the average cost of production $\bar{x} = 11{,}200$

Recall:

$$\bar{x} = \frac{\sum_{i=1}^{n}(f_i \cdot x_i)}{\sum_{i=1}^{n} f_i} \rightarrow N$$

$4 + 2 + a + 3 + b = 20$

$9 + a + b = 20$

$a + b =$

Costs'00(USD)	x_i '00	f_i	$f_i \cdot x_i$ '00
~~6000 - 8000~~ 60 - 80	$\frac{60+80}{2} = 70$	4	$(70 \times 4) = 280$
~~8000 - 10000~~ 80 - 100	90	2	180
100 - 120	110	8	880
120 - 140	130	3	390
* 140 - 200*	170.	3	510
Total		20	2240

$$\bar{x} = \frac{2240}{20} = 112 \times 100$$

11200

$$\bar{x} = \underline{11200 \text{ USD}}$$

Determining the arithmetic mean using an assumed mean [working mean].

✓ Guess
✓ predicting
✓ suppose

Case I: For ungrouped Data.

✓ Here we apply the formular:

$$\bar{x} = A + \frac{\sum(x_i - A)}{N}$$

where;

A = assumed mean
x_i = data set values
N = Number of data set values

Example:

The wages of 5 employees were: 600, 900, 850, 750, 650. Using an assumed mean of 780, determine the average wage.

Recall:

$$\bar{x} = A + \left[\frac{\sum_{i=1}^{n}(x_i - A)}{N}\right]$$

$\boxed{X_a = A}$

x_i	A	$x_i - A$
600	780	−180

900	780	120
850	780	70
750	780	-30
650	780	-130
Total		-150

0

+120

$$\bar{x} = 780 + \left[\frac{-150}{5}\right]$$

$$= 780 - 30 = 750$$

$$\bar{x} = \underline{\underline{750}}$$

Case II: For Grouped Data.

✓ Once the class mark has been obtained, we use the formular:

$$\bar{x} = A + \left[\frac{\sum_{i=1}^{n}\left[f_i \cdot (x_i - A)\right]}{\sum_{i=1}^{n} f_i}\right]$$

where:

A = assumed mean

x_i = class mark

f_i = class frequency

Example:

The units sold by K-Inc were as noted

units	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60

No clients	4	2	1	8	1

Determine the average number of units sold using a working mean of 37 units.

Recall:

$$\bar{x} = A + \left[\frac{\sum[f_i \cdot (x_i - A)]}{\sum f}\right]$$

units	x_i	f_i	A	$(x_i - A)$	$f_i(x_i - A)$
10 – 20	$\frac{10+20}{2}$ 15	4	37	−22	−88
20 – 30	25	2	37	−12	−24
30 – 40	35	1	37	−2	−2
40 – 50	45	8	37	8	64
50 – 60	55	1	37	18	18
	Total	16			−32

$$\bar{x} = 37 + \left[\frac{-32}{16}\right] = 37 - 2$$

$$= 35$$

$$\bar{x} = \underline{\underline{35 \text{ units}}}$$

Determining the arithmetic mean using the stepwise [coding] method

Case I: For ungrouped data.

✓ Here, the arithmetic mean shall be given by:

$$\bar{x} = A + \left[\frac{\sum d}{N}\right] \times h$$

where;

A = assumed mean

$d = \left[\frac{x_i - A}{h}\right]$; N = size of the data set

x_i = data set values

h = common term [divisor]

Example:

Given the costs of production; 600, 800, 900, 1200, 550; with an assumed mean of 850, determine the average cost by the coding method

Recall:

$$\bar{x} = A + \left[\frac{\sum d}{N}\right] \times h$$

$$\Rightarrow d = \frac{x_i - A}{h}$$

x_i	A	h	$(x_i - A)$*	$d = \left(\frac{x_i - A}{h}\right)$
600	850	100	-250	-2.5
800	850	100	-50	-0.5
900	850	100	50	0.5
1200	850	100	350	3.5
550	850	100	-300	-3
			Total	-2

$$\bar{x} = 850 + \left(\frac{-2}{5}\right) \times \frac{100}{1} \quad \left(-\frac{2}{5} \times 100\right)$$

$$= 850 - 40 = 810$$

$$\bar{x} = \underline{810 \text{ USD}}$$

Class III

Friday
July 05, 2024

Case II: For Grouped Data. h

✓ * Here, the common term (factor) may be taken to be the same as the class width; we apply the formular:

$$\bar{x} = A + \left[\frac{\sum_{i=1}^{n}(f_i \cdot d)}{\sum f}\right] \times h$$

where;

A = assumed mean

f_i = class frequency

$d = \dfrac{x_i - A}{h}$

x_i = class mark

h = common term (factor)

Example:

The sales made by X-Inc were taken and recorded as shown:

Sales '00 ugx	50-100	100-150	150-200	200-250	250-300
units	4	8	5	6	2

Determine the average sales with an assumed mean of 100 by the coding (step wise) method

Recall:

$$\bar{x} = A + \left[\frac{\sum(f \cdot d)}{\sum f}\right] \times h$$

$h = 50$

Sales '000 Ugx	f	x	A	$(x_i - A)$	h	$d = \frac{x_i - A}{h}$	f·d
50 - 100	4	$\frac{50+100}{2}$ 75	100	-25	50	-0.5	-2
100 - 150	8	125	100	25	50	0.5	4
150 - 200	5	175	100	75	50	1.5	7.5
200 - 250	6	225	100	125	50	2.5	15
250 - 300	2	275	100	175	50	3.5	7
Total	25						31.5

$$\bar{x} = 100 + \left(\frac{31.5}{25}\right) \times 50$$

$$= 100 + 63 = 163 \times 100$$

$$\bar{x} = \underline{\underline{16300 \text{ UGX}}}$$

The Weighted (mean) Average

✓ Here, a scalar value (weight) is attached to the variable depending on how much value that variable contributes.

- We hence use the formular:

$$\text{Weighted average, } \bar{x} = \frac{\sum_{i=1}^{n} (w_i \cdot x_i)}{\sum_{i=1}^{n} w_i}$$

where;

w_i = Weight of the variable

x_i = variable in Question

Example:

The household expenditure was as summarised:

Item	Weight (units)	Rate (ugx)
Sugar	10	5000
salt	4	800
soap	8	7000
soda	6	12000
Rice	12	6000

Determine the weighted average cost incurred by the house hold.

Recall:

$$(\bar{x})\ \bar{w} = \frac{\sum (w_i \cdot x_i)}{\sum w_i}$$

x_i	w_i	$w_i \cdot x_i$
5000	10	50·000
800	4	3200
7000	8	56000
12000	6	72000
6000	12	72000
Total	40	253200

$$\bar{x} = \frac{253200}{40} = 6330$$

$$\bar{x} = \underline{\underline{6330\ \text{Ugx}}}$$

Expectation, $E(x)$ $\bar{x}$

✓ Given a random variable x, the mean (expectation) of x shall be given by:

✓ Hope
✓ may (NOT)
✓ Probability.
✓ unsure.

$$E(x) = \sum_{i=1}^{n} \left[x_i \cdot p(x) \right] \checkmark\checkmark$$

where: x_i = the data set values

$P(x)$ = probability of variable x

Example:

The checks of five employees were as noted.

Employee	A	B	C	D	E
check (USD)	800	1200	900	700	1600
Expectation, $P(x)$	0.25	0.15	0.30	0.20	0.10

Determine the average pay for the 5 employees.

Recall:

$$E(x) = \sum [x_i \cdot P(x)]$$

x_i	$P(x)$	$x \cdot P(x)$
800	0.25	200
1200	0.15	180
900	0.30	270
700	0.20	140
1600	0.1	160
Total		950

$E(x) = 950$ USD

The Geometric Mean

✓ This kind of the mean is used in situations where changes in the variable are exponential in nature.

✓ It is defined to be the n^{th} root of the product of the data set values; it is given by:

$$G.M = \sqrt[n]{(x_1 \cdot x_2 \cdot x_3 - - - - x_n)}$$

where; $x_i = \{x_1, x_2, - - - x_n\}$ data set values

$n =$ Number of Data set Values

Example:

The number of students that registered for a course was as noted.

Day	Mon	Tues	Wed	Thur	Fri
No of students	40	60	120	25	250

Determine the average no of students using the geometric mean.

Recall:

$$G.M = \sqrt[n]{(x_1 \cdot x_2 \cdot - - - - x_n)}$$

$$G.M = \sqrt[5]{(40 \times 60 \times 120 \times 25 \times 250)}$$

$$= 70.97.$$

$$G.M = 70.97.$$

<u>NB:</u> sometimes logarithms may be used to determine the G.M; hence we use the formular;

$$G.M = \underbrace{\text{anti log}}_{\to \log^{-1}} \left[\frac{\sum \log x_i}{N}\right]$$

Refer to the number of students.

Day	Mon	Tues	Wed	Thur	Fred
students' No	40	60	120	25	250

Determine the Geometric mean.

Recall:

$$G.M = \log^{-1}\left[\frac{\sum \log x_i}{N}\right]$$

x_i	$\log x_i$
40	1.6021

log 40

60	1.7782
120	2.0792
25	1.3979
250	2.3979
Total	9.2553

$$G.M = \log^{-1}\left[\frac{9.2553}{5}\right]$$

1.85106

shift $\boxed{\log}$ ( ) 10^x

$GM = \underline{\underline{70.97}}$

Geometric mean for Grouped / frequency Data

Once the class mark has been obtained, we apply the formular:

$$GM = \log^{-1}\left[\frac{\sum_{i=1}^{n}(f_i \cdot \log x)}{\sum_{i=1}^{n} f_i}\right]$$

where:

f_i = class frequency

x_i = mid-mark

Example:

The cost of production for J-Inc was as summarised

Cost Ugx	60 - 80	80 - 100	100 - 200	200 - 400	400 - 900
№ of units	4	8	2	3	3

Determine the average cost of production using the geometric mean

Recall:

$$G.M = \log^{-1}\left[\frac{\sum(f.\log x)}{\sum f}\right]$$

Cost (Ugx)	x	f	$\log x$	$f.\log x$
60 - 80	70	4	1.8451	7.3804
80 - 100	90	8	1.9542	15.6336
100 - 200	150	2	2.1761	4.3522
200 - 400	300	3	2.4771	7.4313
400 - 900	650	3	2.8129	8.4387
	Total	20		43.2362

$$G.M = \log^{-1}\left[\frac{43.2362}{20}\right] = 145.15$$